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AN EXTENSION OF THE CHAPMAN-FERRARO  
THEORY OF GEOMAGNETIC STORMS

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PREFACE

This Memorandum is part of a continuing theoretical study of fields and particles, sponsored by the National Aeronautics and Space Administration under Contract NASr-21(05).



ABSTRACT

The consequences of an electric field of broad scale across the magnetosphere are outlined. Such an electric field can be induced by the motion of solar streams past the geomagnetic field, and this was suggested by Chapman and Ferraro in their original theory of geomagnetic storms. Plasma trapped in the geomagnetic field will drift adiabatically in this electric field. The dynamics of such plasma motions in the magnetosphere are discussed here. It is shown that the emplacement of the main-phase ring current, local acceleration of auroral primaries and outer zone electrons, and the distribution of fluxes of energetic electrons in the outer magnetosphere are possible consequences of such an electric field. A subsidiary result is that a severe restriction on charge-separation theories of the auroras and ionospheric current systems is eliminated.



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After thirty years, the Chapman-Ferraro theory of geomagnetic storms remains basic to our understanding of geomagnetic disturbances associated with solar activity (Chapman and Ferraro, 1931, 1932, 1933, 1940). Considerable effort has been expended on refining and extending the original theory, and several details have been resolved and clarified. A review of these advances is beyond the scope of the present paper, and the reader is referred to an excellent review paper by Parker (1962). Two problems central to storm theory remain unsolved. First, the mechanism for emplacement of the main-phase ring current is obscure, as is, in fact, the location of the ring current. Second, auroral zone phenomena are clearly related to the growth and strength of the main-phase ring current (Akasofu and Chapman, 1961). However, the mechanism linking the two phenomena is at present not firmly established.

It is clear (Parker, 1962; Akasofu, Chapman and Venkatesan, 1963) that storms with a large main-phase decrease ( $\sim 500 \gamma$ ) in the surface field require a ring current located fairly deep in the geomagnetic field *it*, say, 3--5 earth radii. Hydromagnetic shock waves have been proposed as a means of locally accelerating trapped plasma (Dessler, Hanson and Parker, 1961; Kern, 1962b). As yet, however, no observations support the propagation of such shock waves in the (essentially) collisionless portions of the magnetosphere. The high rates of dumping of energetic particles

from the magnetosphere and the gross changes in particle fluxes observed in association with magnetic storms seem to require a supply mechanism that is operating constantly and is enhanced during a geomagnetic storm (O'Brien, 1962, 1963). In the following we outline a simple extension of the Chapman-Ferraro theory that may supply answers to some of these problems.

In the original development of their theory of geomagnetic storms, Chapman and Ferraro pointed out that a solar stream enveloping the geomagnetic field would become electrically polarized. Positive charges appear on the morning side of the cavity and negative on the evening side, as shown in Fig. 1. This polarization was seen to result from the opposite orbital directions of positive ions and electrons in the geomagnetic field. More recently, this type of polarization has been used in schemes for injecting plasmas into confining magnetic fields in fusion studies (Rose and Clark, 1961). Figure 2 shows the orbits of ions and electrons near the boundaries of a plasma penetrating a magnetic field. It is easily seen that the flow of solar plasma past the geomagnetic field is geometrically the exact inverse of the situation in Fig. 2. The electric field produced by the solar stream moving past the geomagnetic field would be across the interior of the cavity in the stream and directed from the dawn toward the evening side of the cavity. The local electric field at the interface is given simply by  $E = - (1/c) v_{st} \times B$ , where  $v_{st}$  is the component of the stream

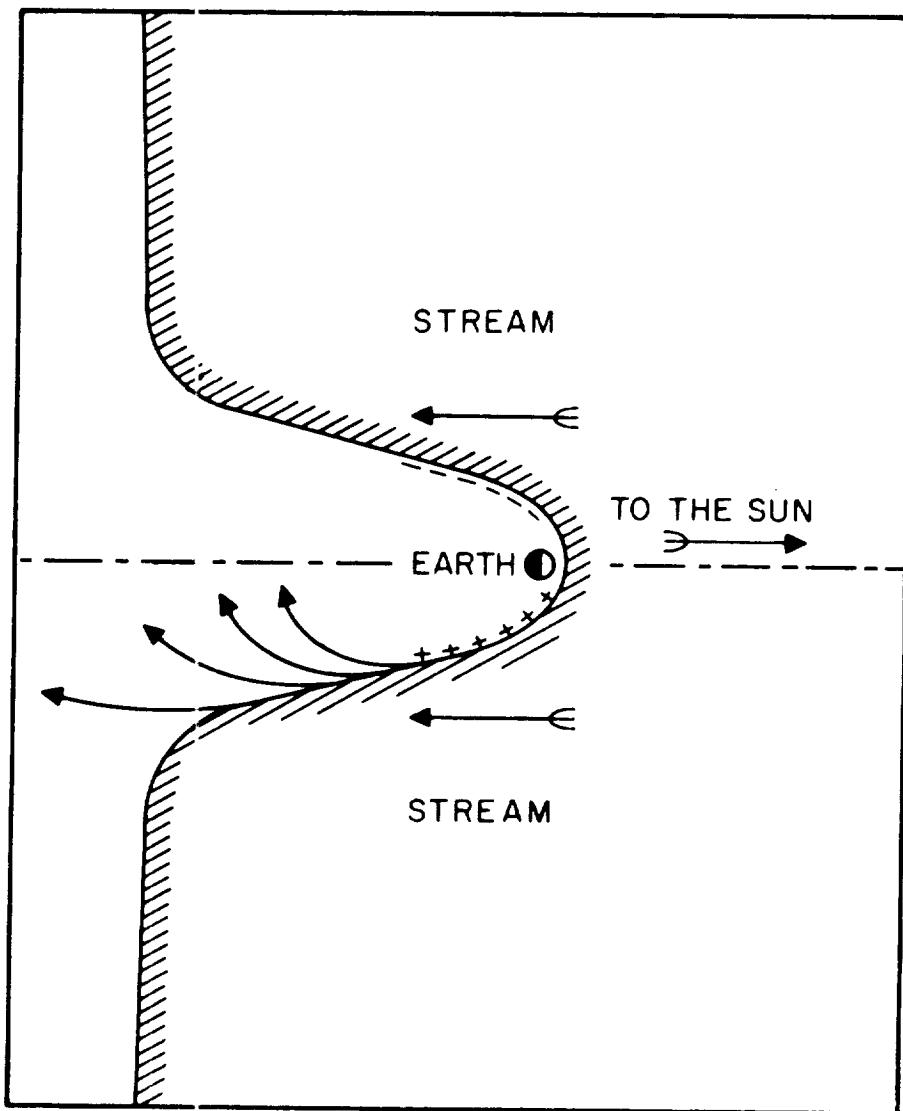


Fig. 1 Equatorial section of magnetospheric boundary  
(after Chapman and Ferraro)

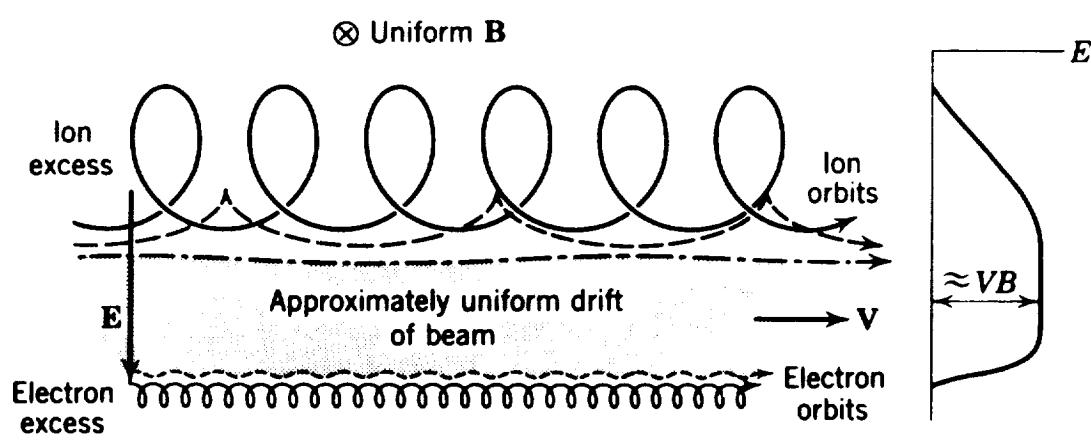


Fig. 2 Schematic representation of typical particle orbits and magnitude of electric field in a dense beam of ions and electrons crossing a magnetic induction  $B$ ; the beam is assumed to be thick in the  $B$ -direction  
(after Rose and Clark)

velocity tangent to the interface,  $c$  is the velocity of light, and  $B$  is the local magnetic field at the interface. Note that this field is independent of  $v_n$ , the component of the stream velocity  $v$  perpendicular to the interface. Ferraro (1952) has shown an additional charge separation field  $E'$  to be associated with  $v_n$ , due to the mass difference between electrons and ions.  $E'$  is everywhere directed antiparallel to  $v_n$  and must be confined to the interface between the stream and magnetic field. Note also that the field  $E$  is perpendicular to  $B$  at the interface. The field  $B$  must be tangent to the interface except at a pair of "null points" slightly upstream from where the geomagnetic axis pierces the interface (Spreiter and Briggs, 1962a). Thus at the interface  $E \cdot B = 0$ , and there is no component of  $E$  along  $B$ . We find that, apart from some perplexing problems of the electric field geometry in the cavity,  $E$  is clearly neither "short-circuited" along  $B$ , nor transferred directly from the cavity walls to the ionosphere along magnetic field lines, as suggested by Cole (1961).

If the cavity were a vacuum, the geometry of the electric field could be specified uniquely by the shape of the boundary, the direction of flow of the solar stream, and the magnetic field inside the boundary. It turns out that the electric stress associated with  $E$  affects the balance of pressure across the interface by a negligible amount, of the order  $v^2/c^2$ , where  $v$  is the stream velocity and  $c$  the velocity of light. The electric field distribution could even in principle be worked out for the

case where magnetic field lines are required to be equipotentials. However, the presence of trapped plasma in the cavity complicates the problem tremendously. First of all, the electric permittivity of the plasma is quite high. Spitzer (1956) and Chandrasekhar (1960) show that a magnetized plasma has an electric permittivity  $\epsilon$  given by  $\epsilon = 1 + 4\pi\rho c^2/B^2$ , where  $c$  is the velocity of light,  $\rho$  the mass density per  $\text{cm}^3$ , and  $B$  is the magnetic field. In terms of the Alfvén velocity,  $v_A$ , of hydromagnetic disturbances,  $\epsilon = 1 + c^2/v_A^2$ . For the magnetosphere and interplanetary space  $v_A \ll c$  and  $\epsilon \approx c^2/v_A^2$ . This of course means that  $\epsilon \gg 1$ . Further, various studies of the variation of  $v_A$  with altitude indicate that  $\epsilon$  will be rather strongly altitude-dependent (MacDonald, 1961). The magnitudes and altitude dependence of  $v_A$  are not reliably known for the outermost parts of the magnetosphere. Since  $\epsilon$  must be considered in determining the electrostatic field's distribution in the cavity, the lack of knowledge of  $v_A$  limits our ability to fully describe the problem at the outset.

A second fundamental difficulty results from the consideration of  $E \times B$  drift of the plasma in the cavity. Bulk motions of the magnetized plasma result from the presence of the electric field  $E$ . The local velocity  $v_E$  of this bulk motion is given by the familiar  $v_E = c E \times B / B^2$ . This motion clearly redistributes the plasma in the cavity, leading to modifications in the plasma density, and hence also to modified distributions of  $v_A$  and  $\epsilon$ . Thus a solution for the electric field across the cavity must be self-consistent with the required  $E \times B$  flow field of the plasma

in the cavity. Solution of this complete problem is obviously quite difficult for even a gross idealization of the geometry of the magnetosphere. However, some immediate conclusions can be drawn from a simplified model.

Consider Fig. 1, after Chapman and Ferraro (1931). The charge distribution shown will correspond qualitatively to the polarization described above, and it leads to an electric field directed across the cavity from the morning side toward the evening side. This electric field persists downstream from the earth. The intensity of the magnetic field decreases downstream, while the solar-plasma velocity tangent to the interface becomes very nearly constant (since the cavity becomes nearly cylindrical). Thus the electric field  $E = - (1/c) \nabla \times B$  must decrease in intensity downstream from the earth, being orthogonal to and nearly proportional to  $B$ . This electric field will produce an earthward bulk motion of the plasma trapped in the "tail" of the magnetosphere, with a local drift velocity  $v_E$  given by  $cE \times B / B^2$ .

In general, the flow of magnetospheric plasma will be toward the sunward part of the cavity. Since the electric field is everywhere orthogonal to the interface, plasma flow never crosses the boundary of the magnetosphere and, near the boundary, always moves toward the front of the cavity. The plasma flow pattern engendered by the electric field therefore has no closed flow loops connected with high latitude field lines inside the cavity. This type of flow differs considerably from flow patterns deduced from ionospheric current systems such as those of Axford and Hines (1961),

Piddington (1962), and Cole (1961). Before discussing the plasma motions or their consequences in greater detail, a few remarks regarding the energetics of this kind of system are appropriate.

As the plasma moves inward from the tail of the magnetosphere, it encounters an increasing magnetic field. For plasma with a finite temperature this implies a compression and an adiabatic heating. The magnetic moments of the individual charged particles in the plasma are conserved during  $\underline{E} \times \underline{B}$  drift motion. Further, the number density  $n$  of the moving plasma is proportional to the local magnetic field, or  $n/B = \text{constant}$ . Since the magnetic moment  $M = W_{\perp}/B$ , where  $W_{\perp}$  is the transverse kinetic energy of a particle and  $B$  is the magnetic field, it follows that  $\alpha_1$ , the ratio of the local density of transverse kinetic energy to the energy density in the magnetic field is constant during the drift motion. The invariance of  $\alpha_1$  during the inward drift motion in the tail of the magnetosphere will be recalled later in connection with the development of the main-phase ring current.

Since the trapped plasma is diamagnetic, we can expect it to resist being moved into the stronger magnetic field by the  $\underline{E} \times \underline{B}$  drift. This is indeed the case. A finite blob of plasma imbedded in the geomagnetic field will become polarized due to the westward drift of positive ions and the eastward drift of electrons (the familiar drifts due to the gradient of  $B$  and curvature of field lines). The direction of the electric field due to this polarization is such as to cause  $\underline{E} \times \underline{B}$  drift of the diamagnetic

plasma outward into a weaker magnetic field. In fact, Chandrasekhar (1960) shows that the drift rate for plasma in a gravity field corresponds to the free gravitational acceleration of the plasma, provided the polarization field can grow without limit. However, the electric field resulting from polarization of a plasmoid imbedded in the geomagnetic field is limited by the finite conductivity of the ionosphere to which the plasma is connected by the very large conductivity along magnetic field lines. In this case the polarization electric field that can develop depends on the height-integrated direct (Pedersen) conductivity and on the total current resulting from the relative east-west drift of ions and electrons in the plasmoid. Put quite simply, the plasma resists the inward motion due to a broad-scale electric field by becoming locally polarized due to a westward (direct, not Hall) current in the ionosphere. The local polarization opposes the component of the broad-scale electric field that causes  $E \times B$  motion into a stronger magnetic field. The local westward current due to relative drift of ions and electrons depends essentially on the local magnetic field (and its geometry) and on the energy density of the plasma. A broad-scale electric field across the magnetospheric cavity will "drive" plasma toward the front of the cavity until the energy density of the plasma increases sufficiently to produce a counter-polarization that locally cancels the driving electric field. Note that the transverse kinetic energy gained by the plasma in the  $E \times B$  drift motion into the stronger magnetic field is balanced by a decrease in the

energy in the over-all electric field. The energetics of individual particle motion in this kind of system have been discussed briefly by Dungey (1963). The westward drift of ions and the eastward drift of electrons in the nonuniform magnetic field (parallel and anti-parallel to the over-all electric field, respectively) occur at rates such that the particles lose electrostatic potential energy at the same rate as they gain kinetic energy.

It was mentioned above that, in the tail of the magnetosphere, the  $\mathbf{E} \times \mathbf{B}$  drift velocity inside the boundary is nearly equal and antiparallel to the solar-stream velocity outside. Plasmoids may enter the magnetosphere through the downstream walls of the cavity, either due to Helmholtz-type instabilities in the cavity wall (Parker, 1958) or due to local continuity or cancellation of the geomagnetic field by interplanetary fields imbedded in the solar plasma. Such "injected" plasma will be accelerated to the inward  $\mathbf{E} \times \mathbf{B}$  drift velocity. Since most of the mass of the plasma is concentrated in the positive ions, and the motions are all adiabatic, this acceleration implies a finite drift of the plasma parallel to the electric field  $\mathbf{E}$ . Both Spitzer (1956) and Chandrasekhar (1960) have discussed this so-called "polarization drift." On the morning side of the cavity, this drift is toward the cavity's interior, away from the wall, hence the acceleration to the sunward  $\mathbf{E} \times \mathbf{B}$  drift will aid injection on this side of the cavity. On the evening side of the cavity, however, the reverse is true: there the "polarization drift" is outward and would

presumably inhibit the injection of solar plasma. The mechanism suggested here may play a significant role in feeding the trapping regions with energetic electrons. If so, we can predict a pronounced asymmetry of electron flux contours relative to the midnight meridian. Figure 3 shows contours for the electron flux distribution on the morning side of the cavity as determined by Frank, Van Allen and Macagno (1963) from data of Explorers XII and XIV. If injection and  $E \times B$  drift of the kind suggested here is operating, similar contours for the evening side will be closer to the earth, since less plasma will enter from this side of the cavity. Note that the injection of plasma by  $E \times B$  drift from the downstream sides of the cavity will occur even during geomagnetically quiet times, as long as the solar wind flows and a supply of plasma crossing the cavity walls is available. The supply may be sufficient to account for high-latitude ionospheric current systems and auroras during magnetically quiet periods.

An enhancement of the velocity of the solar wind leads directly to an enhancement of the electric field across the cavity. Any change in the electric field will propagate as a hydromagnetic disturbance across the cavity in a short time ( $\sim 1$  minute). A concomitant of this enhanced electric field will be an increased drift rate of plasma into the forward part of the cavity. In addition, on the night side, plasma that was previously in or near equilibrium (self-polarized to exclude the quiet-time electric field) will drift deeper into the geomagnetic field. High-latitude geophysical phenomena that are related to plasma motions in the

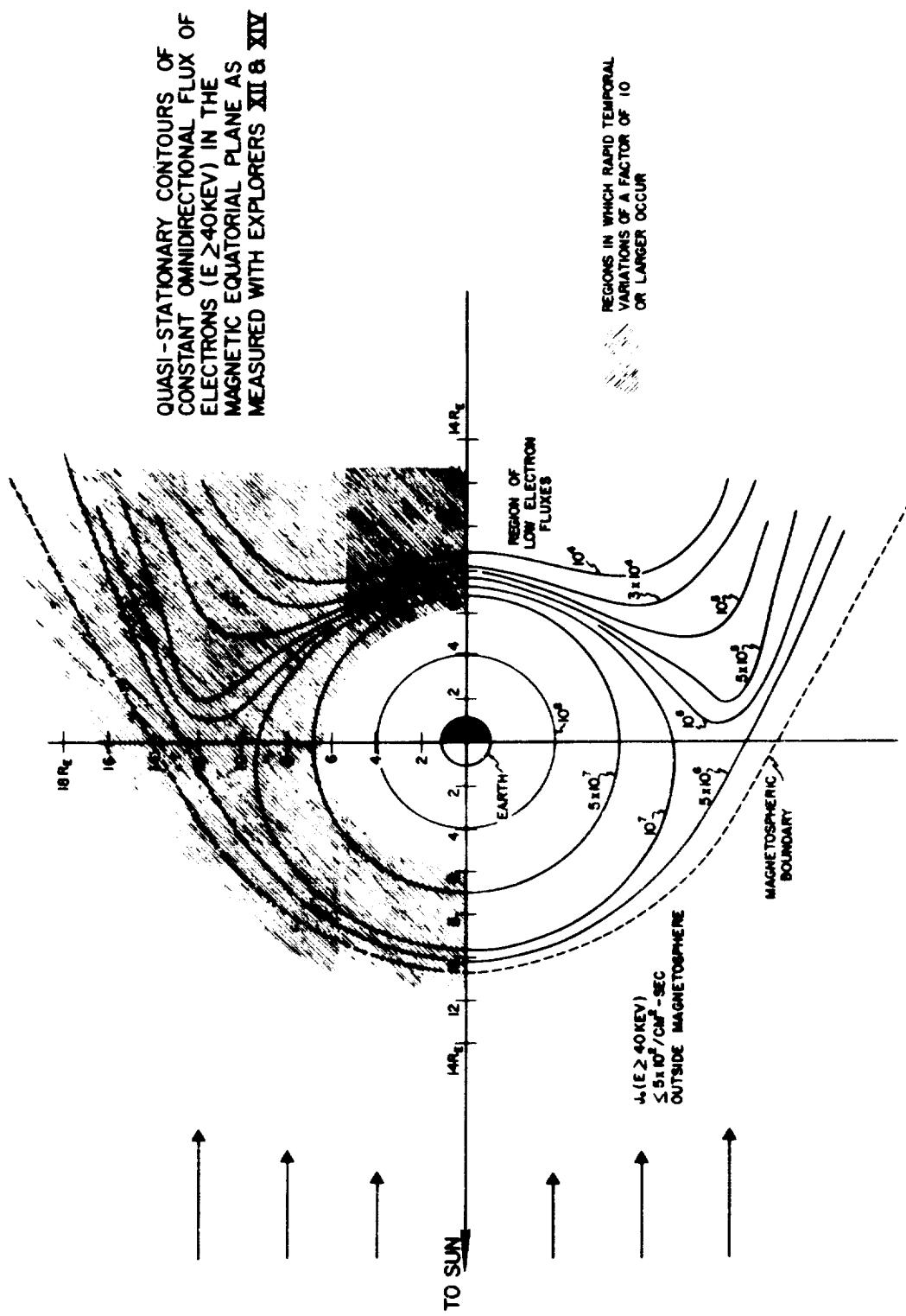


Fig. 3 Quasi-stationary contours of constant omnidirectional flux of electrons ( $E \geq 40$  KeV) in the magnetic equatorial plane as measured with Explorers XII and XIV  
(after Frank, Van Allen, and Macagno)

cavity or to dumping of energetic particles from a trapped plasma will therefore reflect a one-to-one correlation with the velocity of the solar wind. For example, the  $K_p$  index of planetary magnetic activity should vary in the same fashion as the solar wind velocity. This is indeed found to be the case (Snyder and Neugebauer, 1963).

Using this model, the emplacement of the main-phase ring current of a geomagnetic storm can be described as a result of a net transport of trapped plasma deeper into the geomagnetic field by an enhanced electric field across the cavity. Note that an enhanced density of the solar wind will cause an increased compression of the magnetic field in the cavity, as in a sudden commencement. The electric field at the boundary of the cavity will be increased during a density enhancement of the solar wind due to the increased magnetic field at the boundary. This will modify the equilibrium flow of plasma in the cavity, since the field increase throughout the cavity will be, on the average, much less than at the boundary. An enhancement of the stream velocity will, however, be much more effective in modifying the plasma flow. Consider the following example. At the boundary  $E = -(1/c)v_t \times B$ , while  $B^2/8\pi = f\rho v_n^2$ , where  $\rho$  is the mass density of the stream,  $v_t$  and  $v_n$  are the components of the stream velocity  $v$  tangent and normal to the boundary respectively, and  $f$  is a constant between 1 and 2 relating to the character of the flow (Spreiter and Briggs, 1962b). If the orientation of the boundary relative to  $v$  is nearly constant for variations in  $v$  and  $\rho$ , we have  $E \propto v^2 \sqrt{\rho}$ . Hence

a 16-fold increase in  $\rho$  would be required to produce the same four-fold change in  $E$  as a two-fold increase in  $v$ . We have noted above that the acceleration of trapped plasma occurs at the expense of the energy of the local electric field  $E$  in the cavity. Hence a four-fold increase in  $E$ , due to either an enhanced  $v$  or  $\rho$  in the solar-plasma flow, enhances by a factor of 16 the energy available for conversion into plasma kinetic energy. The above discussion indicates that the density and velocity of solar streams have a separate importance in relation to geomagnetic storm phenomena.

The main phase of a geomagnetic storm is thought to result from the local generation or emplacement of an energetic plasma in the geomagnetic field (cf. Parker, 1962). The present theory suggests that this ring current results from the enhancement of the broad-scale electric field described above due to an enhanced velocity or density of the solar plasma flow. If the energy in the broad-scale electric field increases, trapped plasma will move inward on the night side of the magnetosphere, gaining kinetic energy at the expense of the broad-scale electric field. The available energy appears to be sufficient to account for even great storms. The energy density  $\mathcal{E}_E$  in the electric field is  $\epsilon E^2/8\pi$ , where, as noted above,  $\epsilon$  is the permittivity of the plasma. Neglecting many important features of the geometry of the problem, we can estimate the penetration of plasma into the geomagnetic field and the total energy involved. Since  $v_E = cE \times B/B^2$ , and  $\epsilon \approx c^2/v_A^2$ , we have  $\mathcal{E}_E \approx (B^2/8\pi)(v_E^2/v_A^2)$ .

Now  $B^2/8\pi = \epsilon_M$ , the energy density in the magnetic field, and in the outer magnetosphere  $V_A$  decreases outward. It follows that

$\epsilon_E > \epsilon_M$  above the altitude where  $V_A = V_E$ . We have noted above that during  $E \times B$  motion,  $\beta_1$ , the ratio of plasma kinetic energy  $\epsilon_1$  to magnetic field energy  $\epsilon_M$ , is constant. If a plasmoid moves into the field until its self-polarization equals the local value of the broad-scale electric field, then  $\epsilon_1 = \epsilon_E$ . At this point, then  $\beta_1 \epsilon_M = \epsilon_E$ . Since locally  $\epsilon_E = \epsilon_M (V_E^2/V_A^2)$ , it follows that the plasmoid will penetrate the field to an altitude where  $V_A = V_E/\sqrt{\beta}$ . MacDonald (1961) shows variations of Alfvén velocity  $V_A$  with altitude for two models of the exosphere. For values of  $V_E$  of the order of  $5 \times 10^7$  cm/sec, and a  $\beta_1$  of 0.1, a plasmoid could penetrate to within one or two earth radii of the surface on the night side. If an existing trapped plasma, with a  $\beta_1$  of 0.1, is in equilibrium with a quiet-time solar-wind velocity of, say,  $3 \times 10^7$  cm/sec, doubling the solar stream velocity will (as noted earlier) quadruple the electric field  $E$  and the Alfvén velocity at which the plasma reaches equilibrium. Referring to MacDonald's (1961) Fig. 25 we see that quadrupling the equilibrium value of  $V_A$  would result in an inward motion of about  $1\frac{1}{2}$ --2 earth radii on the night side. On the day side the trapped plasma will move only slightly outward, since there the applicable  $E$  is probably much smaller. The value  $\epsilon_M$  varies as  $1/r^6$ , hence a displacement  $\delta r$  changes  $\epsilon_1 = \beta_1 \epsilon_M$  by a factor of  $\delta \epsilon_1 / \epsilon_1 \approx 6 \delta r/r$ . If the bulk of the plasma energy is initially between 2 and 4 earth

radii, the kinetic energy of the trapped plasma will increase approximately by a factor of 3. Hence a quiet-time ring current energetic enough to produce a  $40\gamma$  decrease at the surface (cf. Akasofu, Cain and Chapman, 1962) would undergo an increase in energy sufficient to produce roughly an additional  $120\gamma$  decrease at ground level. This is sufficient to account for the main phase of an "average" magnetic storm. According to this model for a geomagnetic storm, the onset of the main phase should correspond to an increased velocity in the solar stream. The main-phase ring current can therefore be energized by the above mechanism before the solar stream pressure decreases, as suggested by Explorer XII data (Frank, Van Allen and Macagno, 1963).

Note that the finite conductivity of the ionosphere, to which any plasmoid is connected, implies ohmic losses of plasma energy during  $E \times B$  motion. In the E-region of the ionosphere, Hall currents flow orthogonal to any electric field. These Hall currents are antiparallel to the direction of  $E \times B$  drift. The electric fields in the ionosphere, according to the above discussion, result from the superposition of the broad-scale field (due to polarization of the magnetosphere's boundary) and the self-polarization of energetic plasma inside the magnetosphere. The resultant electric field appears in the ionosphere as a result of the high conductivity along  $B$ , and is very nearly orthogonal to  $B$ . Since the magnetic field lines are very nearly electric equipotentials, the same electric field will appear in the ionosphere in both the northern and southern hemispheres. The

ionospheric currents will, however, be proportional to the local conductivity. Hence differences between the current systems conjugate along  $B$  in opposite hemispheres will be observed.

Here we have discussed, in a general fashion, the consequences of a broad-scale electric field across the magnetospheric cavity. A more detailed discussion is possible, based on deductions from ionospheric current and auroral distributions. These topics will be pursued in a subsequent paper. It is sufficient for our present purpose to point out that the emplacement of the main-phase ring current will coincide with the appearance of large-scale ionospheric-current systems due to ohmic losses from the energized plasma. The inward motion of plasma on the night side during the buildup of the main phase will produce an associated southward motion of the auroral zone, since auroral phenomena are here linked to the loss of particles from plasmoids. The decay of the main phase of a geomagnetic storm will begin when the electric field across the magnetosphere decreases. A distribution of energized plasma in equilibrium with the storm-time electric field will move outward, decreasing the total energy of the trapped plasma and, hence, reducing the main-phase decrease in the surface magnetic field. A decreased electric field would imply a reduced solar-stream velocity and pressure. Hence the decay of the main phase should occur at the same time the magnetosphere's boundary is moving outward as a result of a reduced momentum flux in the solar stream.

Electric fields resulting from charge separation in trapped plasmoids have been discussed by Chamberlain (1961) and Kern (1962a) as the cause of auroras and electrojet currents. The broad-scale electric field discussed above enables us to discard a central objection to such theories, namely, the fact that the self-polarization field will blow a plasmoid out of the geo-magnetic field before significant discharge can occur (Cole, 1962; Chamberlain, 1962). As discussed above, the self-polarization field can at best only halt the inward motion of a plasmoid on the night side of the magnetosphere. Broad-scale dumping of energetic particles can be accounted for either by electric fields along  $B_{\text{m}}$  due to charge separation (Kern, 1962a) or by gradual lowering of mirror points due to inward  $E \times B$  drift (Dungey, 1963). It might be remarked that the preferential injection of plasma from the downstream wall of the cavity on the dawn side favors a relatively larger supply of energetic plasma on the morning side of the magnetosphere. This feature of the theory fits the observed greater intensities of auroras and ionospheric currents during the early morning hours.

O'Brien (1962) suggests that energetic electrons trapped in the outer zone of the Van Allen radiation belts are produced by the same acceleration mechanism as auroral electrons. Thus the trapped electrons are seen as produced by a "splash-back" of electrons that are accelerated but not dumped during ionospheric or auroral bombardment. The view that auroras are produced by precipitation of trapped electrons is therefore an oversimplification of the

processes involved. Electric fields along magnetic field lines are capable of providing both auroral bombardment and simultaneous accelerations of electrons and ions. The energy for such electric fields can come from the kinetic energy of a trapped plasma that is distributed nonuniformly with respect to the adiabatic draft trajectories of charged particles in the geomagnetic field (Kern, 1962a). In the present context, such distributions of energetic plasma are a consequence of injection and acceleration of solar plasma by  $E \times B$  drift in the broad-scale polarization field. An elaboration of these ideas by deductions from the distributions of auroras and ionospheric current systems is beyond the scope of the present paper. It is sufficient to point out that the Chapman--Ferraro polarization of the solar wind provides a common source for both "trapped" and "dumped" energetic particles. Local accelerations of charged particles occur as results of  $E \times B$  drift of plasmoids into a stronger magnetic field and of spatial non-uniformities of such energized plasmoids leading to transient electric fields along magnetic field lines.



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